



Let's solve

$$[x'(t) = -3x(t) + 4y - 9z(t), y' = 6x(t) - y, z'(t) = 10x(t) + 4y + 3z(t)]$$

- Define vector

$$\vec{x}(t) = \begin{bmatrix} x(t) \\ y \\ z(t) \end{bmatrix}$$

- Convert system into a vector equation

$$\vec{x}'(t) = \begin{bmatrix} -3 & 4 & -9 \\ 6 & -1 & 0 \\ 10 & 4 & 3 \end{bmatrix} \cdot \vec{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- System to solve

$$\vec{x}'(t) = \begin{bmatrix} -3 & 4 & -9 \\ 6 & -1 & 0 \\ 10 & 4 & 3 \end{bmatrix} \cdot \vec{x}(t)$$

- Define the coefficient matrix

$$A = \begin{bmatrix} -3 & 4 & -9 \\ 6 & -1 & 0 \\ 10 & 4 & 3 \end{bmatrix}$$

- Rewrite the system as

$$\vec{x}'(t) = A \cdot \vec{x}(t)$$

- To solve the system, find the eigenvalues and eigenvectors of  $A$

- Eigenpairs of  $A$

$$\begin{bmatrix} -\frac{(4726+306\sqrt{291})^{\frac{1}{3}}}{3} + \frac{170}{3(4726+306\sqrt{291})^{\frac{1}{3}}} - \frac{1}{3}, \\ \left( -\frac{11(4726+306\sqrt{291})^{\frac{1}{3}}}{3} + \frac{1870}{3(4726+306\sqrt{291})^{\frac{1}{3}}} + \frac{412}{3} + 2 \left( -\frac{(4726+306\sqrt{291})^{\frac{1}{3}}}{3} + \frac{170}{3(4726+306\sqrt{291})^{\frac{1}{3}}} \right) \right)^{\frac{1}{3}}, \\ 2 \left( -\frac{11(4726+306\sqrt{291})^{\frac{1}{3}}}{3} + \frac{1870}{3(4726+306\sqrt{291})^{\frac{1}{3}}} + \frac{412}{3} + 2 \left( -\frac{(4726+306\sqrt{291})^{\frac{1}{3}}}{3} + \frac{170}{3(4726+306\sqrt{291})^{\frac{1}{3}}} \right) \right)^{\frac{1}{3}} \end{bmatrix}$$

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- Consider eigenpair

$$\begin{bmatrix} -\frac{(4726+306\sqrt{291})^{\frac{1}{3}}}{3} + \frac{170}{3(4726+306\sqrt{291})^{\frac{1}{3}}} - \frac{1}{3}, \\ \left( -\frac{11(4726+306\sqrt{291})^{\frac{1}{3}}}{3} + \frac{1870}{3(4726+306\sqrt{291})^{\frac{1}{3}}} + \frac{412}{3} + 2 \left( -\frac{(4726+306\sqrt{291})^{\frac{1}{3}}}{3} + \frac{170}{3(4726+306\sqrt{291})^{\frac{1}{3}}} \right) \right)^{\frac{1}{3}}, \\ 2 \left( -\frac{11(4726+306\sqrt{291})^{\frac{1}{3}}}{3} + \frac{1870}{3(4726+306\sqrt{291})^{\frac{1}{3}}} + \frac{412}{3} + 2 \left( -\frac{(4726+306\sqrt{291})^{\frac{1}{3}}}{3} + \frac{170}{3(4726+306\sqrt{291})^{\frac{1}{3}}} \right) \right)^{\frac{1}{3}} \end{bmatrix}$$